Name: Solutions

> This assignment is worth 100 points. You will be awarded 40 points for attempting the entire assignment (that is answer all problems). All problems will be graded for the remaining 60 points. The space left between each question is indicative of how much work you should show. If there are any problems you find particularly difficult, circle them in red. If there are any particular questions you would like feedback on, circle them in green. These are examples of questions that might appear on an exam/quiz. If you use a calculator to help, make sure you can also do them without it.

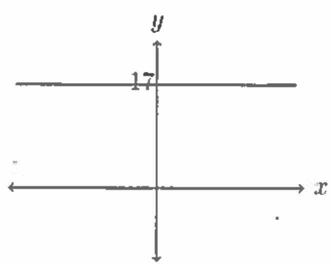
In this homework we will look at how to graph functions. Each question give's a description of each kind of function and what properties their graphs have. Use this as a guide to graph the functions given after.

When graphing functions you will be graded on 3 things;

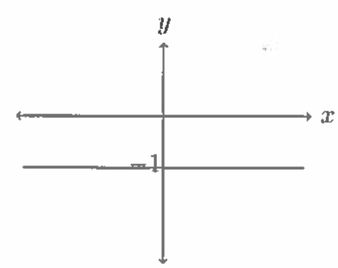
- Shape of the graph
- Labeling the point of intersection with the y-axis (i.e. $(0, y(0))$)
- Labeling the point(s) of intersection with the x-axis

1. Constant Functions: The simplest of functions. A constant function is one that always outputs the same value, no matter what the input. The domain is all real numbers.

   Example: $y(x) = 17$

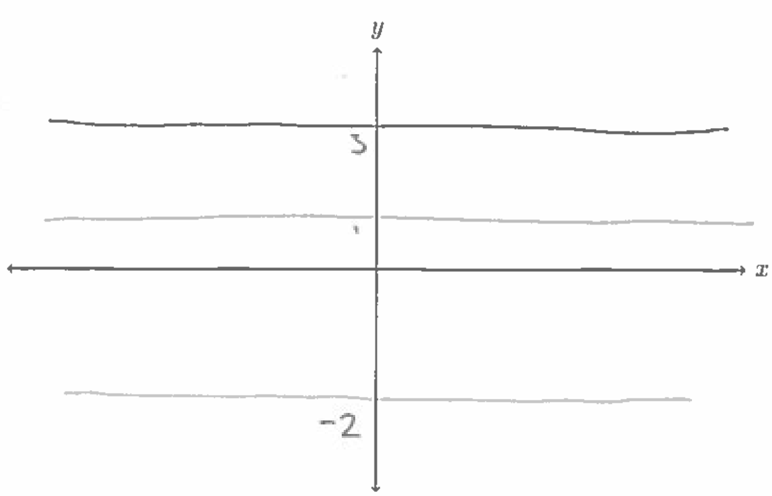
   

   Example: $y(x) = -1$

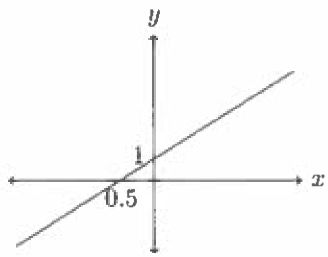
   - Each of these graphs is a single, horizontal line.
   - Each of them cross the y-axis at $(0, c)$ where c is the constant $f(x)$ is equal to.
   - If $y(x) = 0$, then the graph is the x-axis. If $y(x) \neq 0$ then the graph never crosses the x-axis.

   On the axis below, draw the graphs $y = -2$, $y = 1$ and $y = 3$. Label each function and all points of intersection.

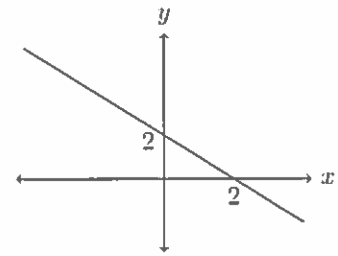
   

2. Linear Functions: They're like constant functions, but slanted. A linear function is one of the form $y = mx + c$, where m is the *slope* and c is the y-intercept.

Example: $y(x) = 2x + 1$



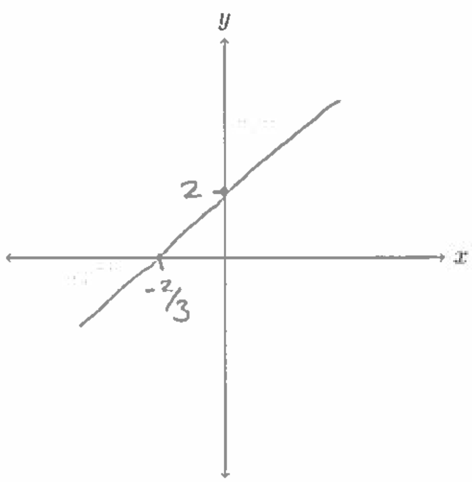
Example: $y(x) = -x + 2$



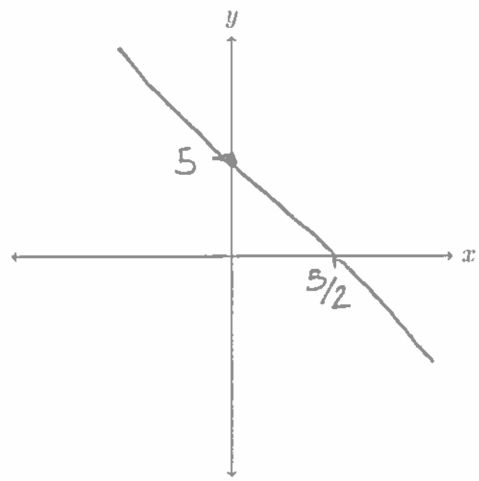
- Each of these graphs is a single, *straight* line. If $m > 0$, the line goes from bottom left to top right. If $m < 0$, the line goes from top left to bottom right.
- Each of them cross the y-axis at $(0, c)$ where c is the intercept value given in $y(x) = mx + c$ is equal to.
- $f(x)$ crosses the x-axis exactly once, at the point $\frac{-c}{m}$.

On the axis below, draw the given linear function. Include all points of intersection with coordinate axes.

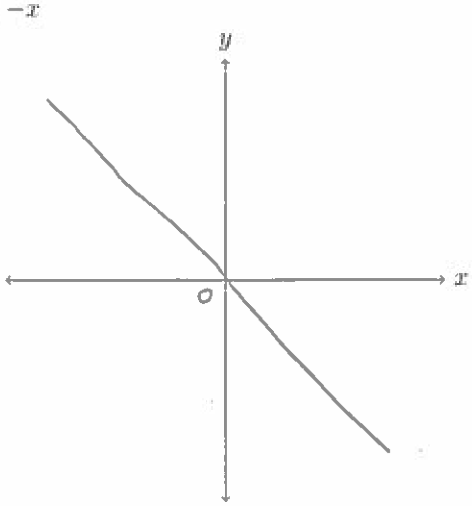
(a) $3x + 2$



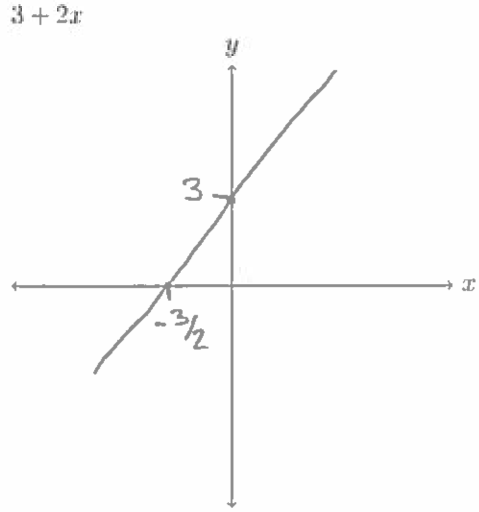
(c) $5 - 2x$



(b) $-x$

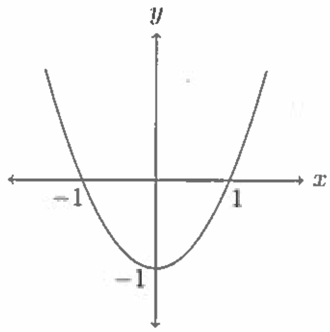


(d) $3 + 2x$

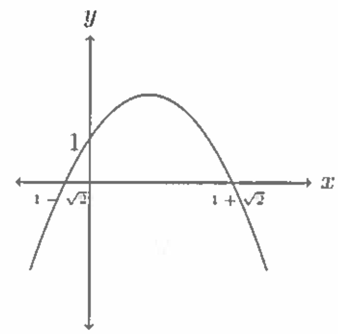


3. Quadratic Functions: They're not straight lines. A quadratic function is one of the form $y = ax^2 + bx + c$. They sort of look u-shapes, called *parabolas*.

Example: $y(x) = x^2 - 1$



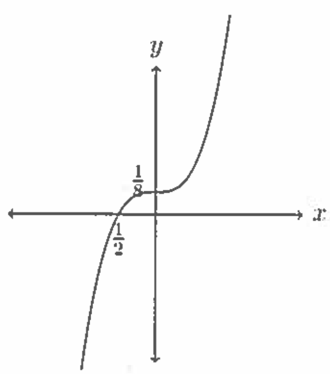
Example: $y(x) = -x^2 + 2x + 1$



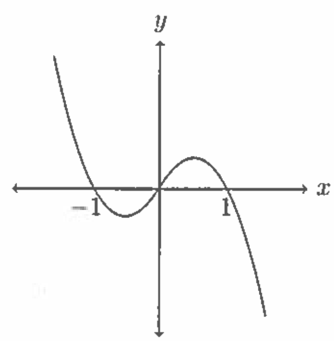
- Each of these graphs is a parabola. If $a > 0$, it looks like a u, i.e. the tails are going up. If $a < 0$, it looks like an n, i.e. the tails are going down.
- Each of them cross the y-axis at $(0, c)$ where c is the intercept value given in $y(x) = ax^2 + bx + c$ is equal to.
- $f(x)$ can cross the axis $0, 1$ or 2 times, depending on how many roots there are. Remember you can check the discriminant to find this out.

4. Cubic Functions: A cubic function is one of the form $y = ax^3 + bx^2 + cx + d$. They look like a wiggly line.

Example: $y(x) = x^3 + \frac{1}{8}$



Example: $y(x) = -x^3 + x$

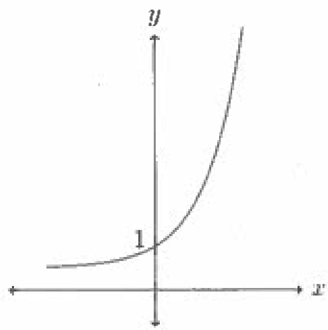


- Each of these graphs is a wiggly line, with at most two points where it changes direction. If $a < 0$, it goes from the top left to the bottom right. If $a > 0$ it goes from the bottom left to the top right.
- Each of them cross the y-axis at $(0, d)$ where d is the intercept value given in $y = ax^3 + bx^2 + cx + d$.
- Cubics *always* cross the x-axis at least once. They have up to 3 points of intersection with the x-axis.

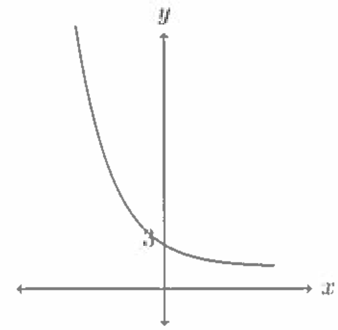
Cont.

5. <u>Exponential Functions</u>: An exponential function is one of the form $y = p \cdot a^{mx}$. They look like a boomerang. A key feature is that they blow up really fast.

Example: $y(x) = 2^x$



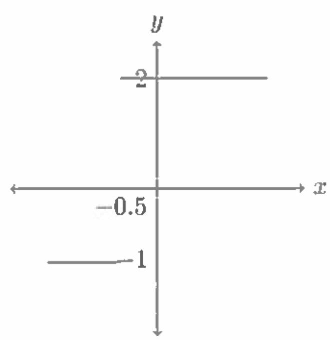
Example: $y(x) = 3 \cdot 5^{-x}$



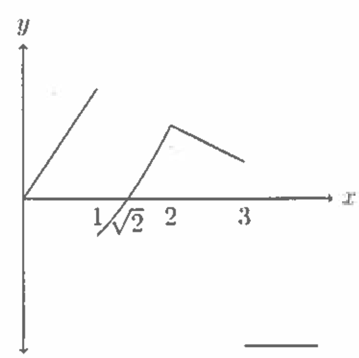
- Each of these graphs looks like one of the ones above. If $m > 0$, the function increases. If $m < 0$ the function decreases. If $p > 0$ the function is entirely positive, if $p < 0$ the function is entirely negative.
- Each of them cross the y-axis at $(0, p)$ where p is the coefficient in $y(x) = p \cdot a^{mx}$ is equal to.
- These functions never cross the x-axis.

6. <u>Piecewise Functions</u>: Piecewise functions are just lots of mini graphs of the other types of functions. They can be connected or disconnected, but they never overlap.

Example: $y(x) = \begin{cases} -1, & \text{if } x < -0.5 \\ 2, & \text{if } -0.5 \leq x \end{cases}$



Example: $y(x) = \begin{cases} 3x, & \text{if } x < 1 \\ x^2 - 2, & \text{if } 1 \leq x < 2 \\ 4 - x, & \text{if } 2 \leq x < 3 \\ -4, & \text{if } 3 \leq x \end{cases}$

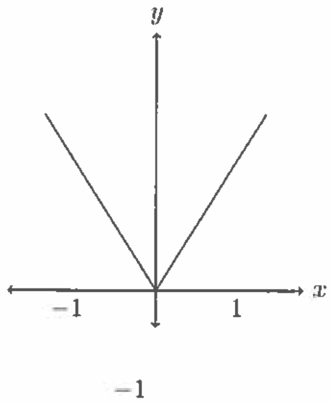


- Each of these graphs have many different parts. They will consist of sections of different functions depending on each pieces domain.
- Each of them cross the y-axis at $f(0)$.
- The x-intercepts are completely determined by the pieces of the function.

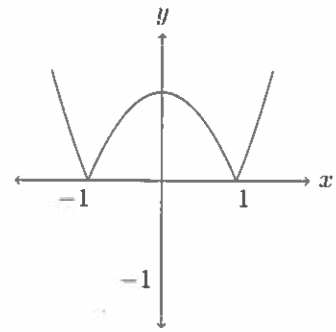
7. Absolute Value Functions: Absolute value functions have corners. They are a special kind of piecewise functions of the form $m \cdot |f(x)|$ where

$$m \cdot |f(x)| := \begin{cases} m \cdot f(x), & \text{if } f(x) \geq 0 \\ -m \cdot f(x), & \text{if } f(x) < 0 \end{cases}$$

Example: $y(x) = |x|$



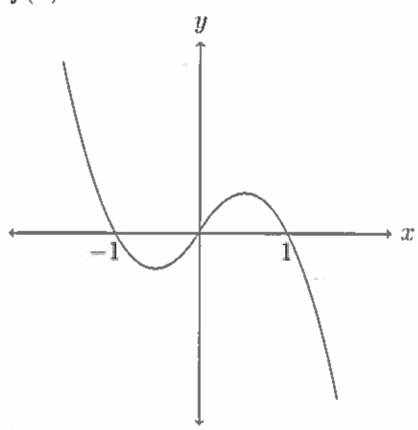
Example: $y(x) = -|x^2 - 1|$



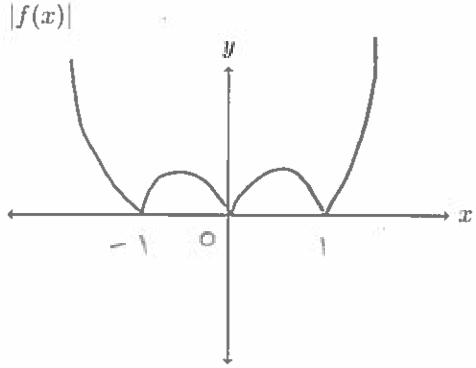
- If $m > 0$, then the function is entirely positive. If $m < 0$, the function is entirely negative.
- Each of them cross the y-axis where $f(x)$ cross the y-axis.
- They *touch* the x-axis at the points $f(x) = 0$, but they never cross the x-axis.

For each of the given functions, draw the corresponding absolute value function.

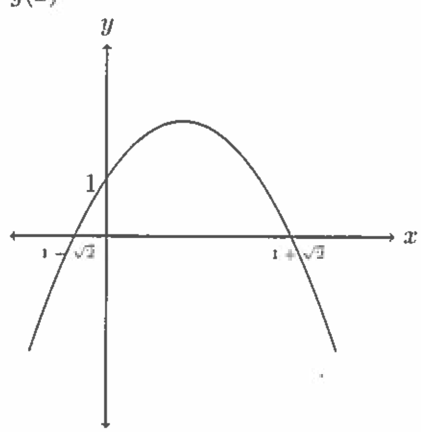
(a) $f(x)$



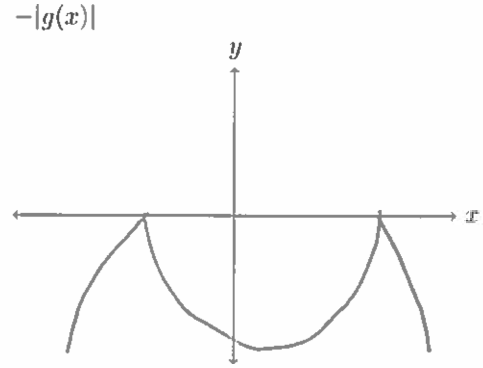
$|f(x)|$



(b) $g(x)$



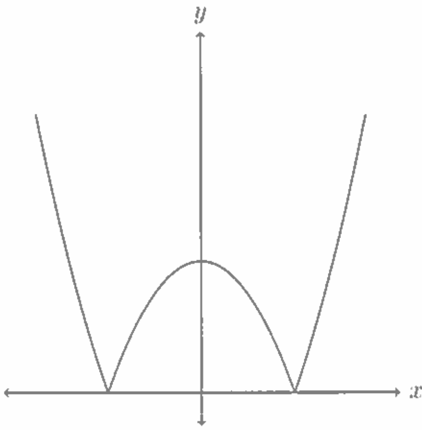
$-|g(x)|$



Cont.

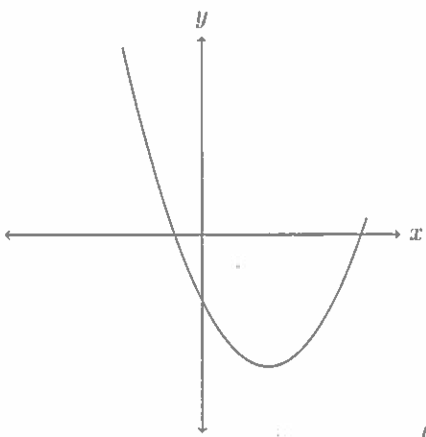
8. For each of the graphs below, decide what type of function they represent based on the descriptions given previously.

(a)



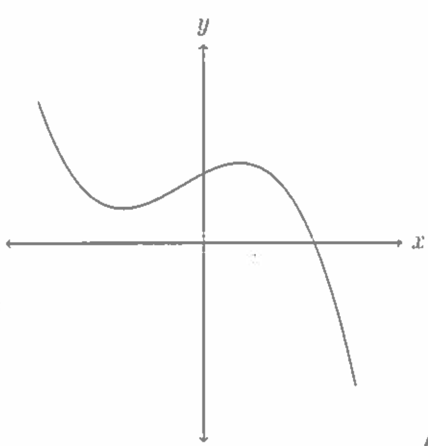
Answer: Absolute

(b)



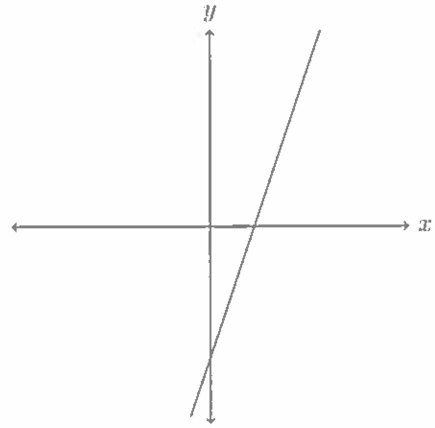
Answer: Quadratic

(c)



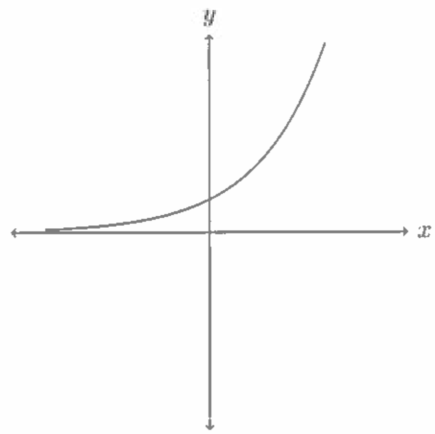
Answer: Cubic

(d)



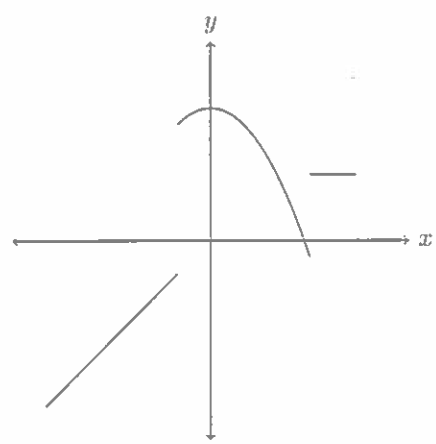
Answer: linear

(e)



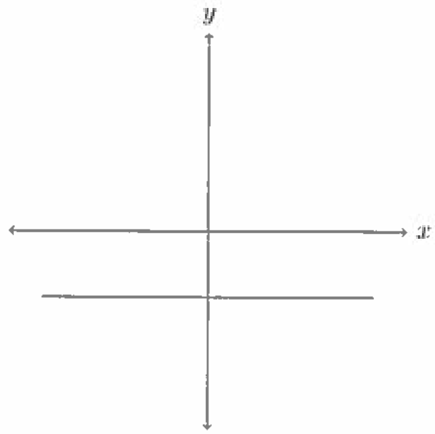
Answer: Exponential

(f)



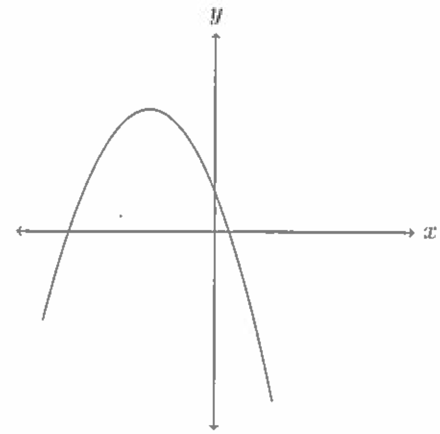
Answer: Piecewise

(g)



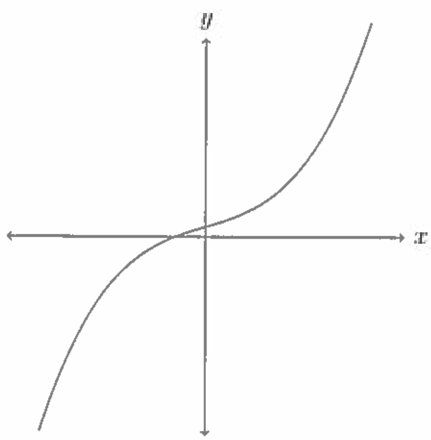
Answer: Constant

(j)



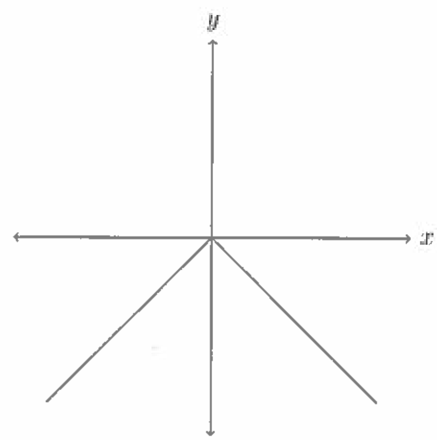
Answer: Quadratic

(h)



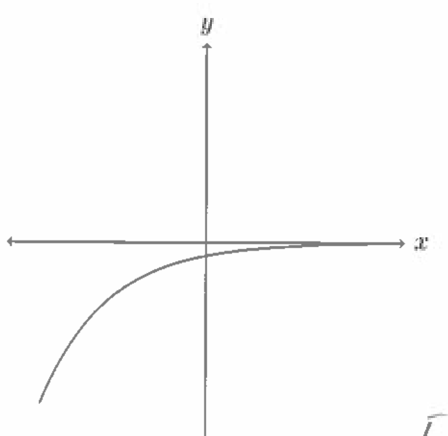
Answer: Cubic

(k)



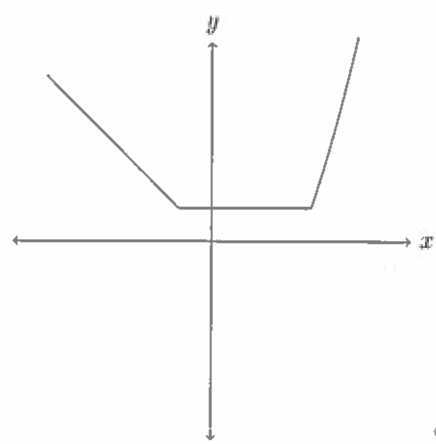
Answer: Absolute

(i)



Answer: Exponential

(l)



Answer: Piecewise

The End.